# MATH 457- Calculus on Manifolds

**Course Code :** 2360457

**Course Credit** :3

**Course Catalog Description**

Review of differentiation, inverse and implicit function theorems, integration on subsets of Euclidean space, tensors, differential forms, integration on chains, integration on manifolds. Stokes` theorem.

**Prerequisites**

Students must have preferably completed Math 251-Math 252 and Math 261-Math 262 (or their equivalents) sequences. If these prerequisites are not met, then the instructor's consent is needed.

**Schedule**

Monday: 13:40-15:30, M103

Wednesday, 08:40 - 09:30, M104

**Course web page**

https://metuclass.metu.edu.tr

**Instructor Information**

**Name/Title :** Prof.Dr. MUSTAFA TURGUT ÖNDER

**Office Address :** Department of Mathematics, Room 140, METU, 06800 Ankara

**Email:** onder@metu.edu.tr

**Office Phone:** 210 5392

**Office Hours**

To be fixed after the first meeting

**Instructional Methods**

- Online lectures using Zoom
- Exercise sets assigned regularly
- Online office hours and discussions

**Course Objectives**

By the end of the course the student will learn

- vector field and differential form concepts on Euclidean spaces
- the meaning of integration on chains in Euclidean spaces
- the manifold concept, the manifold with boundary concept
- vector field and differential form concepts on manifolds
- the meaning of integration on manifolds, Stokes' Theorem on manifolds

**Tentative Weekly Outline**

Week /Topic

1 Review of differentiability and derivatives of maps between Euclidean spaces (Chapters 1 and 2)

2 Review of differentiability and derivatives of maps between Euclidean spaces (cont.) (Chapters 1 and 2)

3 Review of Inverse and Implicit Function theorems (Chapter 2)

4 Tensors, tensor product, alternating tensors (Chapter 4)

5 Wedge product, orientation on vector spaces, volume element (Chapter 4)

6 Tangent space of a Euclidean space at a point, vector fields and differential forms on Euclidean space (Chapter 4)

7 Pull-back and differential of differential forms, statement of Poincaré Lemma (Chapter 4)

8 Brief review of integration on Euclidean spaces, singular cubes and chains in Euclidean spaces (Chapter 4)

9 Integration of forms on chains, Stokes' Theorem on chains (Chapter 3 and 4)

10 Manifolds and manifolds with boundary (Chapter 5)

11 Tangent space of a manifold at a point, vector fields and forms on manifolds (Chapter 5)

12 Differential and pull-backs of forms on manifolds, orientation of manifolds (Chapter 5)

13 Integration on manifolds (Chapter 5)

14 Stokes' Theorem on manifolds (Chapter 5)

**Course Textbook:**

"Calculus on Manifolds" by Michael Spivak, W.A. Benjamin Inc., 1965

**Reading(s)**

Sections of the book indicated above

**Other**

- "A Geometric Approach to Differential Forms" by David Bachman
- "Differential Topology" by Victor Guillemin & Allan Pollack (Chapters 1 and 4).

**Assessment of Student Learning**

**Exams:** There will be three short ((approximately 50 minutes) online midterm exams which will be of weight 20% each, and an online final exam of weight 40%. The final exam will be given in two parts each lasting approximately 50 minutes with a 30-40 minutes break in between. The letter grade will be asssigned by the instructor according to the distribution of weighed total grades.

Exam dates to be fixed in the first two weeks once the composition of the students becomes clear.

**Course Grading**

The weights of the exams are as follows:

Midterm Exam 1 20 %

Midterm Exam 2 20 %

Midterm Exam 3 20 %

Final Exam 40 %

**Course Policies**

**Class Attendance**

Unless there are serious reasons, each student is expected to attend at least 70 percent of the lectures. The instructor should be informed and his consent should be obtained in advance for exceptional cases.

**Make up for Exams and Assignments**

A makeup exam will be given only if there exist a legitimate excuse which can be documented. The final exam will be given at the end of the semester and and cover all the material.